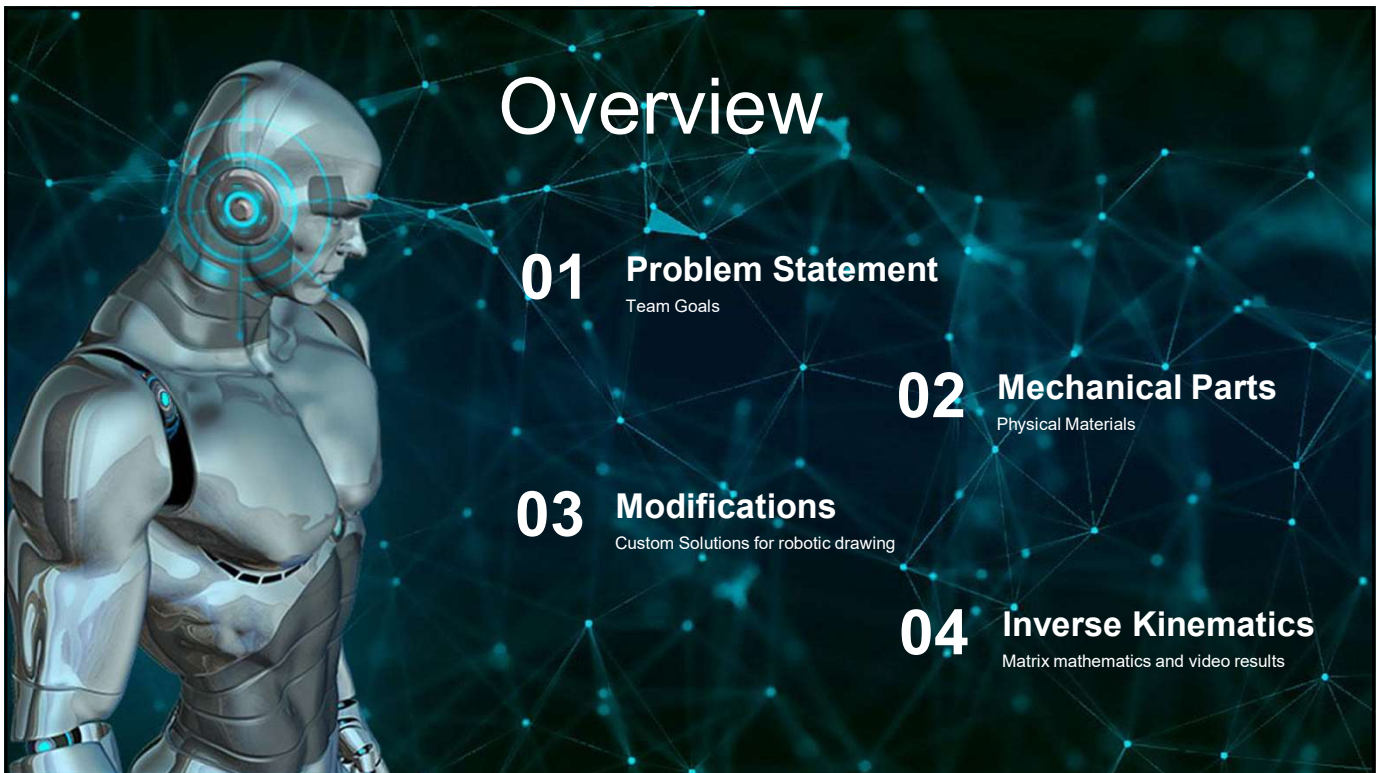




Robot Arm for Drawing

Robotics Final Project



Overview

01 Problem Statement
Team Goals

02 Mechanical Parts
Physical Materials

03 Modifications
Custom Solutions for robotic drawing

04 Inverse Kinematics
Matrix mathematics and video results

Problem Statement

Design and create a robot that can draw simple pictures.



Mechanical Parts

HiWonder Xarm 1s

Intelligent Bus Servo Robotic Arm for Programming

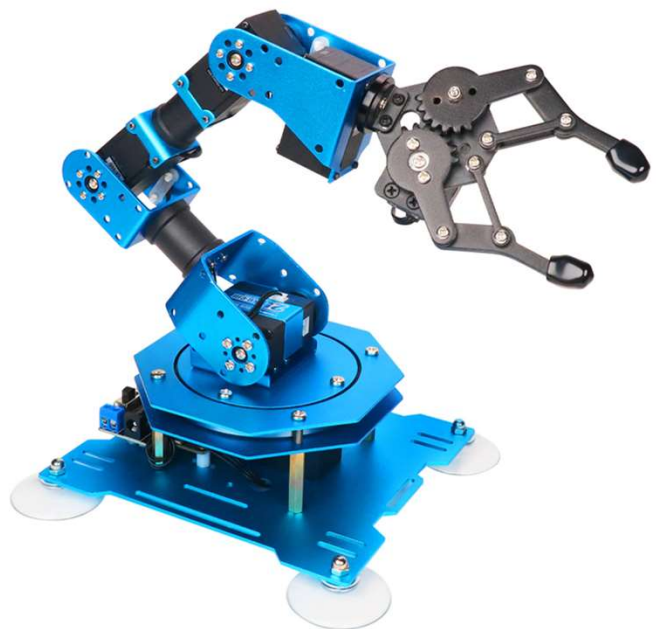
- 6 Degrees of Freedom (including end-effector)
- 5 Aluminum Links
- 6 high torque bus servos



This arm comes with programmable software, and remote control capabilities



This arm is available out-of-the-box from Amazon.



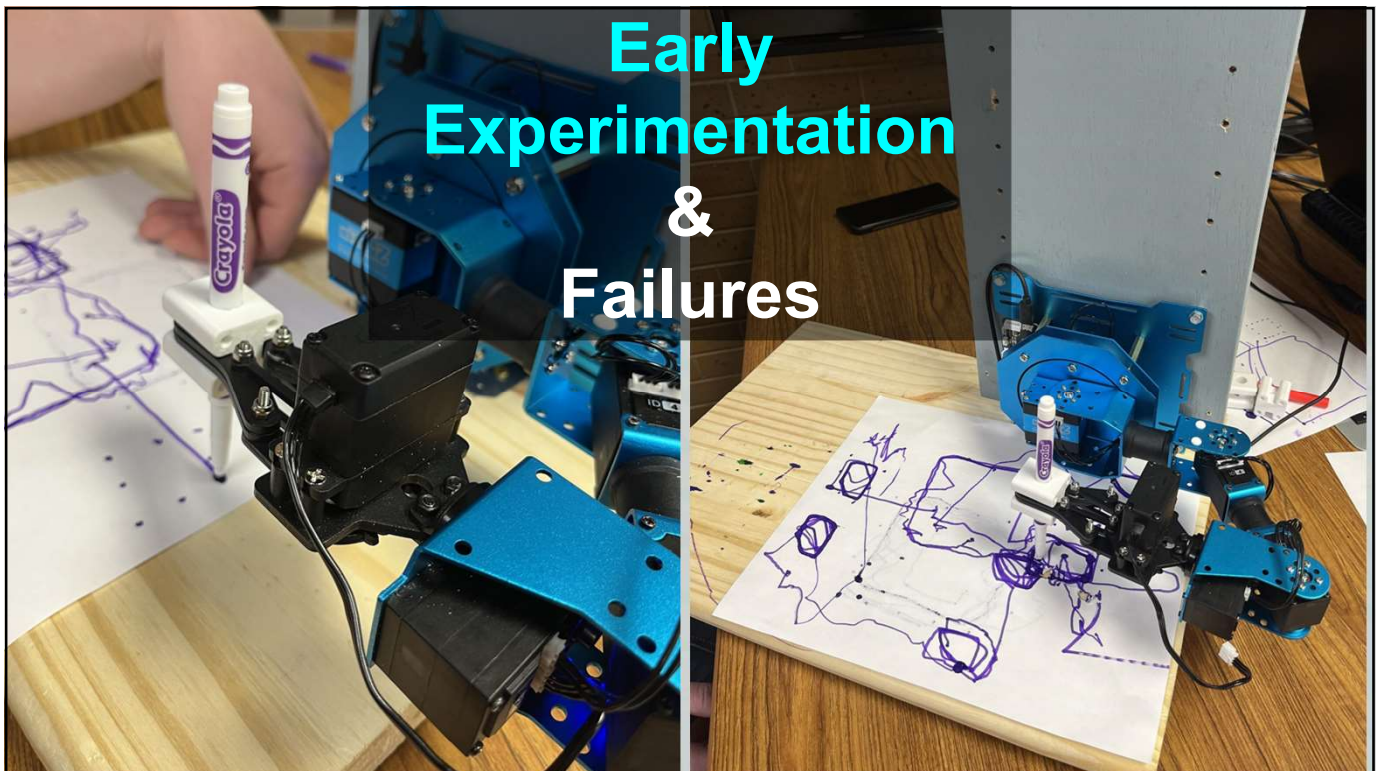
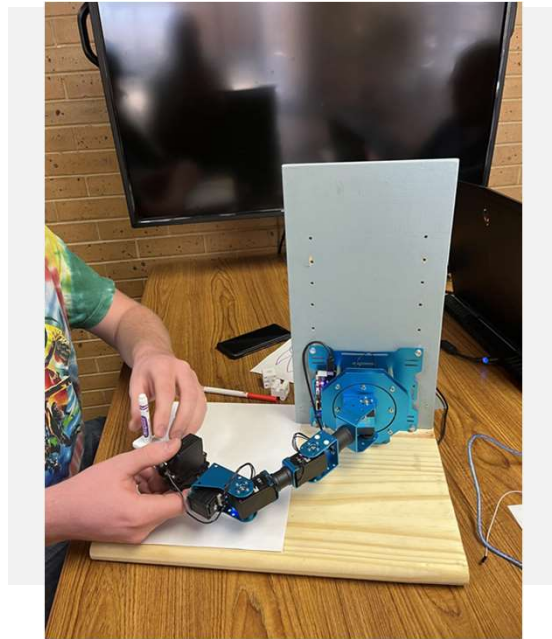
Modifications Mounting Cradle

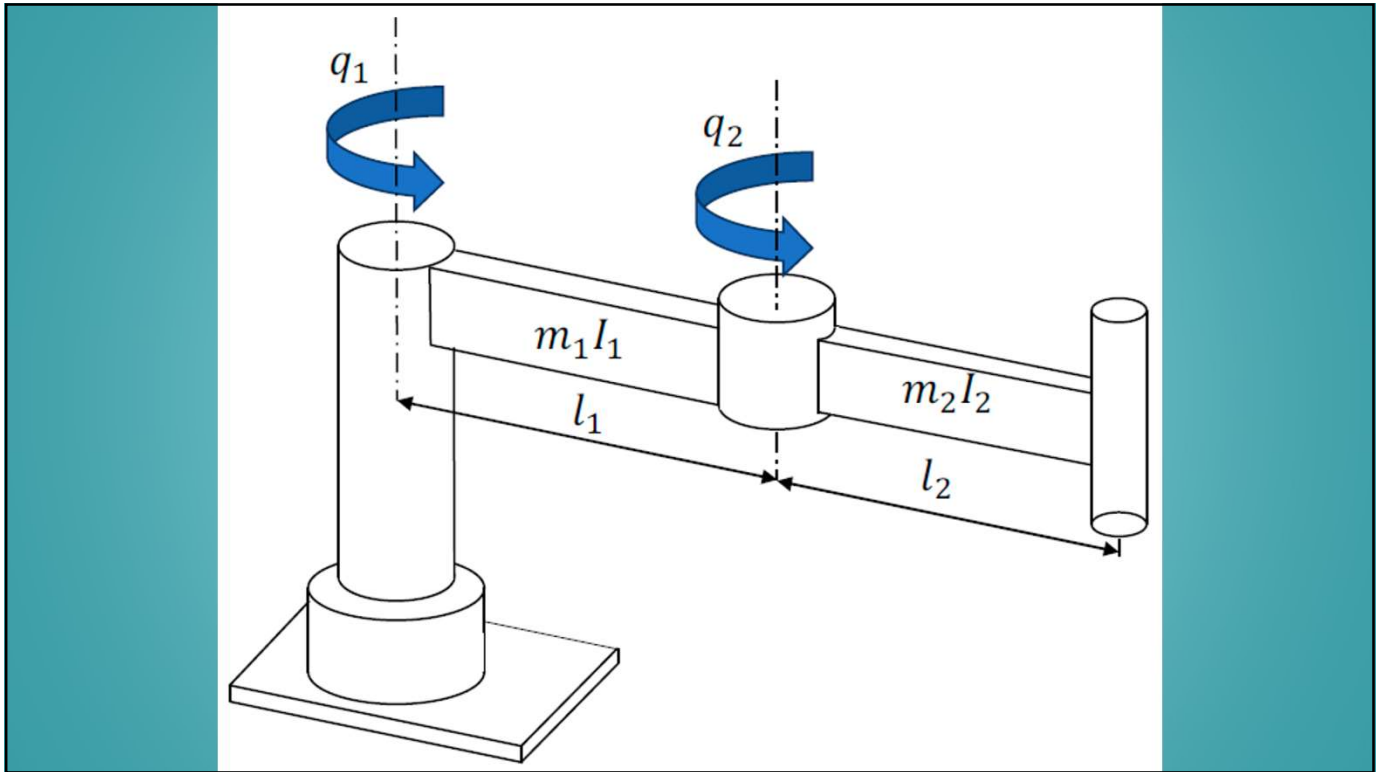
Vertical Mount

We custom built this vertical mount cradle out of wood

It includes holes for adjustable height

The cradle holds the robot vertically so the robot can draw on a horizontal paper





DH Table

i	a	α	d	θ
1	0	0	0	θ_1
2	L_1	0	0	θ_2
3	L_2	0	0	0

$L_1 = 101 \text{ mm}$
 $L_2 = 75 \text{ mm}$

DOF $N=3, J=2, m=3$
 $DOF = m(N-1) - J + \sum F$
 $= 3(3-1) - 2 + 2(1) = 2$

Workspace
 (relative to {0})

${}^0T = R_z \theta_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^1T = R_z \theta_2 + \text{trans } L_1 = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^2T = (\text{no rotation}) + \text{trans } L_2 = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

${}^0T = {}^0T {}^1T {}^2T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} c_1 c_2 - s_1 s_2 & -c_1 s_2 - s_1 c_2 & 0 & L_1 c_1 \\ c_2 s_1 + c_1 s_2 & -s_1 s_2 + c_1 c_2 & 0 & L_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Denavit-Hartenburg Parameters from
(2 DOF) 2 bar linkage

$${}^0_3T = {}^0_2T {}^2_3T = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - c_2s_1 & 0 & L_1c_1 & 1 & 0 & 0 & L_2 \\ c_2s_1 + c_1s_2 & -s_1s_2 + c_1c_2 & 0 & L_1s_1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - c_2s_1 & 0 & L_1c_1 & 1 & 0 & 0 & L_2 \\ c_2s_1 + c_1s_2 & -s_1s_2 + c_1c_2 & 0 & L_1s_1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$x = L_1(c_1c_2 - s_1s_2) + L_1c_1 = L_1c_1 + L_2c_1c_2 - L_2s_1s_2$
 $y = L_1(c_2s_1 + c_1s_2) + L_1s_1 = L_1s_1 + L_2c_2s_1 + L_2c_1s_2$
 $z = 0$

$\rightarrow x^2 + y^2 = (L_1c_1 + L_2c_1c_2 - L_2s_1s_2)^2 + (L_1s_1 + L_2c_2s_1 + L_2c_1s_2)^2$
 $\rightarrow x^2 + y^2 = L_1^2c_1^2 + 2L_1L_2c_1^2c_2 - 2L_1L_2c_1s_1s_2 + L_2^2c_2^2c_1^2 + L_2^2c_2^2s_1^2 + L_2^2s_1^2c_1^2 + L_2^2s_1^2s_2^2 + L_2^2c_2^2s_1^2 + 2L_1L_2c_2s_1s_2 + 2L_1L_2c_1s_1s_2 + L_2^2c_2^2s_1^2 + L_2^2c_1^2s_2^2 + L_2^2s_1^2c_2^2 + L_2^2s_1^2s_2^2$
 $\rightarrow x^2 + y^2 = L_1^2(s_1^2 + c_1^2) + 2L_1L_2c_1^2(s_2^2 + c_2^2) - 2L_1L_2c_1s_1s_2 + 2L_1L_2c_1s_1s_2 + L_2^2c_2^2(s_2^2 + c_2^2) + L_2^2s_1^2(s_2^2 + c_2^2) - 2L_1L_2c_2s_1s_2 + 2L_1L_2c_1s_1s_2$
 $\rightarrow x^2 + y^2 = L_1^2 + 2L_1L_2c_2 + L_2^2c_2^2 + L_2^2s_1^2$

4

$\rightarrow x^2 + y^2 = L_1^2 + 2L_1L_2c_2 + L_2^2(s_2^2 + c_2^2)$
 $\rightarrow x^2 + y^2 = L_1^2 + 2L_1L_2c_2 + L_2^2$
 $\rightarrow c_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2} \rightarrow \theta_2 = \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$
 \rightarrow Plug in $L_1=101$ and $L_2=75$:
 $\rightarrow \theta_2 = \cos^{-1}\left(\frac{x^2 + y^2 - 15826}{15150}\right)$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$ but $\alpha = \theta_1 + \beta$ or $\theta_1 = \alpha - \beta$

$a = L_2s_2, c = L_2c_2, d = L_1$
 $b = c + d = L_1 + L_2c_2$
 $\beta = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{L_2s_2}{L_1 + L_2c_2}\right)$

\rightarrow Plug 0 & 2 into 2: $\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{L_2s_2}{L_1 + L_2c_2}\right)$
 \rightarrow Plug in $L_1=101$ and $L_2=75$:
 $\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{75s_2}{101 + 75c_2}\right)$

Transformation Matrix from DH Table

θ_2 in terms of x and y

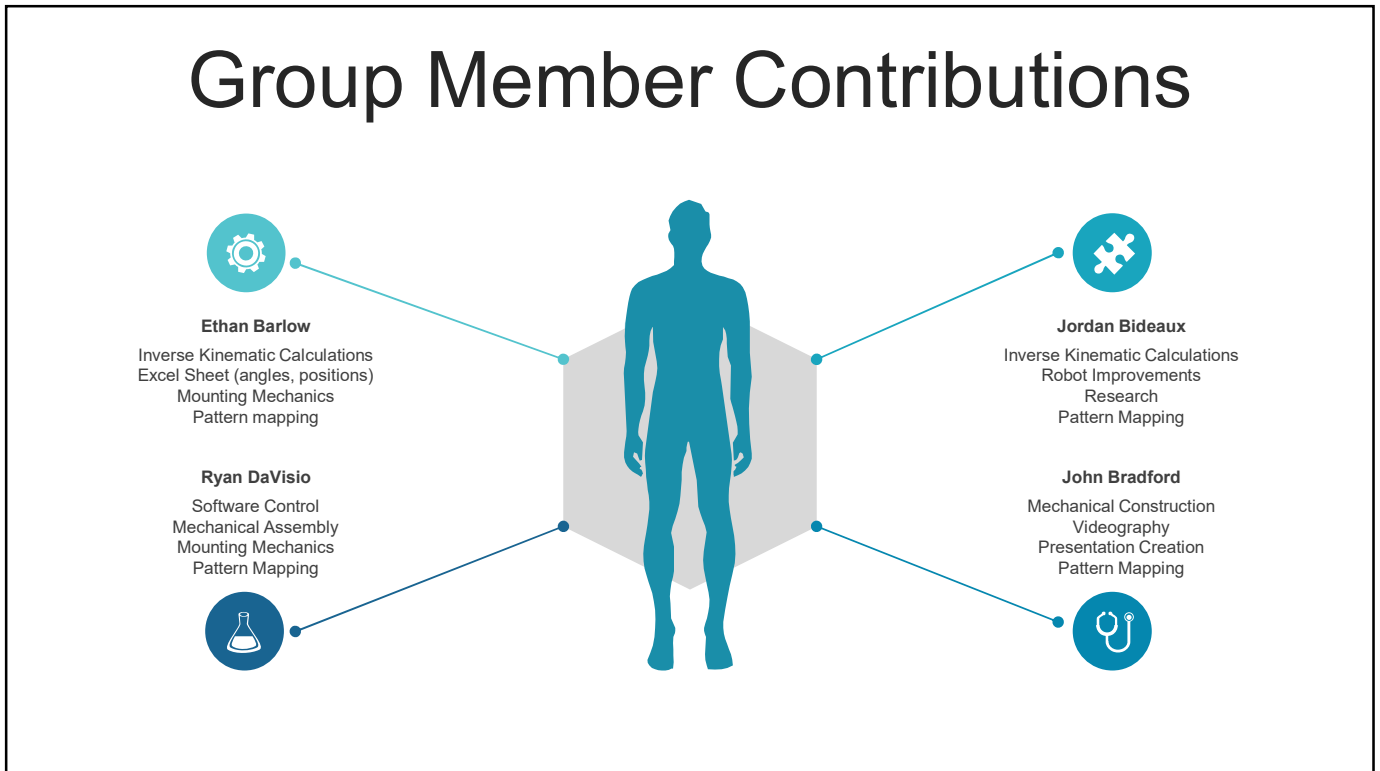
θ_1 in terms of x and y

Angle Conversions

$\theta_{1s} = \frac{1000}{180} (180^\circ - \theta_{1d})$

$\theta_{2s} = \frac{1000}{240} (90^\circ + \theta_{1d})$

Author	Shape	x	y	theta1 (rad)	theta2 (rad)	theta1 (deg)	theta2 (deg)	theta1 (servo)	theta2 (servo)
Jordan	Diamond	25.00	125.00	0.7447	1.5428	42.7	88.4	763	743
		26.56	129.69	0.7708	1.4585	44.2	83.6	755	723
		28.13	134.38	0.7996	1.3700	45.8	78.5	745	702
		29.69	139.06	0.8314	1.2766	47.6	73.1	735	680
		31.25	143.75	0.8666	1.1769	49.6	67.4	724	656
		35.94	145.31	0.8601	1.1214	49.3	64.3	726	643
		40.63	146.88	0.8570	1.0607	49.1	60.8	727	628
		45.31	148.44	0.8574	0.9941	49.1	57.0	727	612
		50.00	150.00	0.8620	0.9203	49.4	52.7	726	595
		48.44	145.31	0.8124	1.0425	46.5	59.7	741	624
		46.88	140.63	0.7683	1.1530	44.0	66.1	755	650
		45.31	135.94	0.7283	1.2549	41.7	71.9	768	675
		43.75	131.25	0.6917	1.3502	39.6	77.4	780	697
		39.06	129.69	0.7006	1.4038	40.1	80.4	777	710
		34.38	128.13	0.7125	1.4536	40.8	83.3	773	722
		29.69	126.56	0.7272	1.4999	41.7	85.9	769	733
		30.00	135.00	0.7943	1.3513	45.5	77.4	747	698
		25.00	135.00	0.8229	1.3698	47.1	78.5	738	702
		20.00	135.00	0.8532	1.3850	48.9	79.4	728	706
		15.00	130.00	0.8482	1.4849	48.6	85.1	730	730
10.00	125.00	0.8499	1.5775	48.7	90.4	729	752		
5.00	120.00	0.8581	1.6634	49.2	95.3	727	772		
10.00	120.00	0.8183	1.6584	46.9	95.0	740	771		
15.00	120.00	0.7800	1.6502	44.7	94.5	752	769		
10.00	115.00	0.7885	1.7366	45.2	99.5	749	790		
5.00	110.00	0.8039	1.8176	46.1	104.1	744	809		
10.00	110.00	0.7603	1.8125	43.6	103.8	758	808		
15.00	110.00	0.7181	1.8040	41.1	103.4	771	806		
10.00	105.00	0.7336	1.8863	42.0	108.1	766	825		
5.00	100.00	0.7566	1.9637	43.4	112.5	759	844		
10.00	100.00	0.7084	1.9584	40.6	112.2	775	843		
15.00	100.00	0.6616	1.9495	37.9	111.7	789	840		
20.00	105.00	0.6465	1.8655	37.0	106.9	794	820		
25.00	110.00	0.6387	1.7769	36.6	101.8	797	799		
30.00	115.00	0.6378	1.6833	36.5	96.4	797	777		
25.00	115.00	0.6728	1.7016	38.5	97.5	786	781		
20.00	115.00	0.7097	1.7166	40.7	98.4	774	785		
25.00	120.00	0.7081	1.6237	40.6	93.0	775	763		
30.00	125.00	0.7131	1.5246	40.9	87.4	773	739		
25.00	125.00	0.7447	1.5428	42.7	88.4	763	743		
20.00	125.00	0.7781	1.5577	44.6	89.2	752	747		
25.00	130.00	0.7828	1.4584	44.9	83.6	751	723		
30.00	135.00	0.7943	1.3513	45.5	77.4	747	698		
John	Triangle	12.00	80.00	0.5996	2.2302	34.4	127.8	809	907
		15.00	95.00	0.6353	2.0198	36.4	115.7	798	857
		18.00	110.00	0.6935	1.7973	39.7	103.0	779	804
		21.00	125.00	0.7713	1.5550	44.2	89.1	754	746
		24.00	140.00	0.8707	1.2796	49.9	73.3	723	680
		27.00	125.00	0.7318	1.5359	41.9	88.0	767	742
		30.00	110.00	0.6019	1.7584	34.5	100.8	808	795
		33.00	95.00	0.4740	1.9574	27.2	112.1	849	842
		36.00	80.00	0.3421	2.1372	19.6	122.5	891	885
		30.00	80.00	0.4001	2.1685	22.9	124.2	873	893
		24.00	80.00	0.4627	2.1946	26.5	125.7	853	899
		18.00	80.00	0.5295	2.2153	30.3	126.9	831	904
		30	90	0.4655	2.0382	26.7	116.8	852	862
		40	90	0.3821	1.9871	21.9	113.9	878	849
		50	90	0.3108	1.9230	17.8	110.2	901	834
		60	90	0.2524	1.8466	14.5	105.8	920	816
		70	90	0.2071	1.7584	11.9	100.8	934	795
		70	100	0.2999	1.6320	17.2	93.5	905	765
		70	110	0.3933	1.4932	22.5	85.6	875	731
		70	120	0.4894	1.3394	28.0	76.7	844	695
70	130	0.5912	1.1655	33.9	66.8	812	653		
60	130	0.6168	1.2572	35.3	72.0	804	675		
50	130	0.6529	1.3326	37.4	76.4	792	693		
40	130	0.6986	1.3934	40.0	79.8	778	708		
30	130	0.7528	1.4401	43.1	82.5	760	719		
30	120	0.6748	1.6055	38.7	92.0	785	758		
30	110	0.6019	1.7584	34.5	100.8	808	795		
30	100	0.5325	1.9020	30.5	109.0	831	829		
Ryan	Square	20	90	0.5597	2.0755	32.1	118.9	822	870
		25	90	0.5114	2.0586	29.3	118.0	837	866
		30	90	0.4655	2.0382	26.7	116.8	852	862
		35	95	0.4580	1.9477	26.2	111.6	854	840
		40	100	0.4578	1.8535	26.2	106.2	854	817
		40	105	0.4960	1.7837	28.4	102.2	842	801
		40	100	0.4578	1.8535	26.2	106.2	854	817
		35	115	0.6049	1.6617	34.7	95.2	807	772
		30	120	0.6748	1.6055	38.7	92.0	785	758
		25	120	0.7081	1.6237	40.6	93.0	775	763
		20	120	0.7432	1.6386	42.6	93.9	763	766
		15	115	0.7483	1.7283	42.9	99.0	762	788
		10	110	0.7603	1.8125	43.6	103.8	758	808
		10	105	0.7336	1.8863	42.0	108.1	766	825
		10	100	0.7084	1.9584	40.6	112.2	775	843
15	95	0.6353	2.0198	36.4	115.7	798	857		
Ryan	Octagon	30	90	0.5597	2.0755	32.1	118.9	822	870
		25	90	0.5114	2.0586	29.3	118.0	837	866
		30	90	0.4655	2.0382	26.7	116.8	852	862
		35	95	0.4580	1.9477	26.2	111.6	854	840
		40	100	0.4578	1.8535	26.2	106.2	854	817
		40	105	0.4960	1.7837	28.4	102.2	842	801
		40	100	0.4578	1.8535	26.2	106.2	854	817
		35	115	0.6049	1.6617	34.7	95.2	807	772
		30	120	0.6748	1.6055	38.7	92.0	785	758
		25	120	0.7081	1.6237	40.6	93.0	775	763
		20	120	0.7432	1.6386	42.6	93.9	763	766
		15	115	0.7483	1.7283	42.9	99.0	762	788
		10	110	0.7603	1.8125	43.6	103.8	758	808
		10	105	0.7336	1.8863	42.0	108.1	766	825
		10	100	0.7084	1.9584	40.6	112.2	775	843
15	95	0.6353	2.0198	36.4	115.7	798	857		



Video of Robot

Demonstrating Forward and Inverse Kinematics

<https://youtu.be/WYeJUNJecsM>